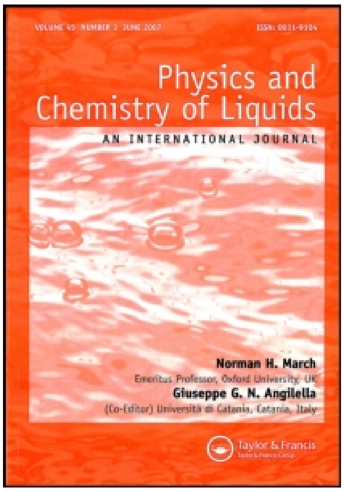

Physics and Chemistry of Liquids
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713646857

Model of R Space Boson-Fermion Mixture and Its Relevance to High-T_C Cuprates

M. L. Chiofalo[a]; M. P. Tosi[a]; N. H. March[a]
[a] Istituto Nazionale di Fisica della Materia and Classe di Scienze, Pisa, Italy

To cite this Article Chiofalo, M. L. , Tosi, M. P. and March, N. H.(1999) 'Model of R Space Boson-Fermion Mixture and Its Relevance to High-T_C Cuprates', Physics and Chemistry of Liquids, 37: 5, 547 — 564
To link to this Article: DOI: 10.1080/00319109908035937
URL: http://dx.doi.org/10.1080/00319109908035937

Phys. Chem. Liq., 1999, Vol. 37, pp. 547–564
Reprints available directly from the publisher
Photocopying permitted by license only

MODEL OF r SPACE BOSON–FERMION MIXTURE AND ITS RELEVANCE TO HIGH-T_c CUPRATES

M. L. CHIOFALO, M. P. TOSI* and N. H. MARCH

Istituto Nazionale di Fisica della Materia and Classe di Scienze, Scuola Normale Superiore, I-56126 Pisa, Italy

(Received 7 April 1998)

Motivated by a correlation between experimentally measured in-plane resistivity R and nuclear spin-lattice relaxation time T_1 for Cu nuclei in an underdoped high-T_c cuprate, a model is set up and solved for the equilibrium between (e) Fermion monomers and (2e) composite r space Bosons. Quantum statistics is fully included and of special interest for the R-T_1 correlation are the numbers of Fermion monomers in equilibrium with these composite Bosons for $T > T_c$. The model is shown to give the gist of the explanation of a pronounced minimum in a plot of the product RT_1 *vs* T for the underdoped cuprate. Some contact is also made with transverse plasmon measurements, which are related to the composite Boson density in the condensate below T_c. Refinements of the simple model used here will eventually need to treat the finite lifetime of the composite Bosons and the screening of the charged particles, especially in the normal state. These can be expected to reduce the temperature range over which, in the normal state, the composite Boson number density is an appreciable fraction of the Fermion monomer density.

Keywords: High-temperature superconductivity; Fermi liquid; real-space charged Bosons

1. INTRODUCTION

Some of the motivation for the present theoretical study has been provided by the work of Egorov and March [1] on an underdoped cuprate investigated experimentally by Bucher *et al.* [2]. In essence, Egorov and March exposed a strong correlation between two

* Corresponding author.

properties in the normal state, namely the in-plane electrical resistivity R and the nuclear spin-lattice relaxation time T_1 of copper nuclei. By using experiment for both quantities, they constructed a plot of the product RT_1 *vs* T which is reproduced in Figure 1. It can be seen that, as the transition temperature T_c is approached from a relatively high temperature, a rather extended linear region, which can be extrapolated natrually enough to pass through the origin of coordinates (not actually shown on the scale of Fig. 1), is followed by a pronounced minimum.

In subsequent work by March, Pucci and Egorov [3], the departure from linearity was interpreted as due to an attractive interaction

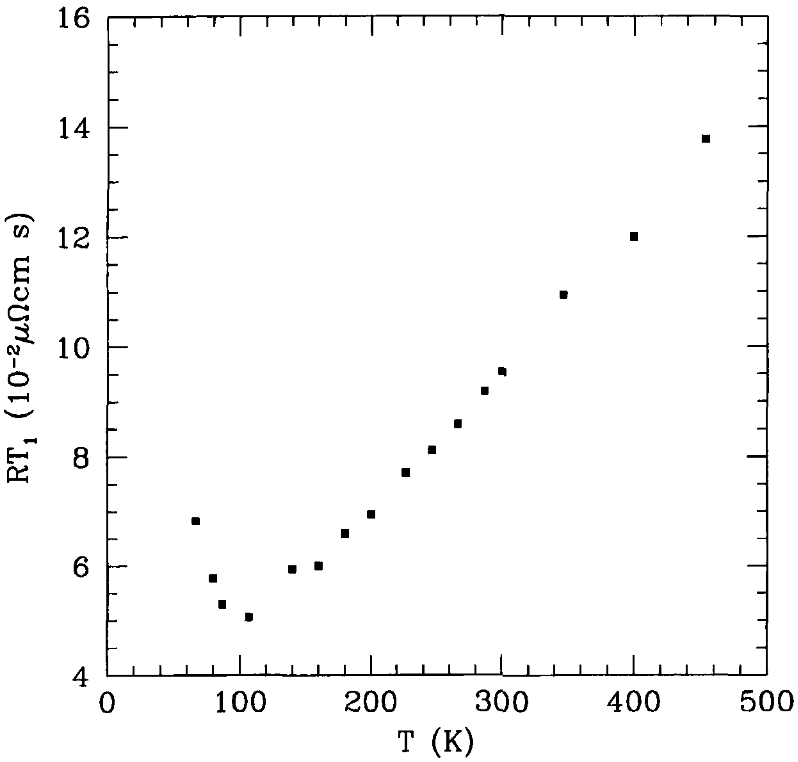

FIGURE 1 Product of RT_1 *versus* temperature T for high-T_c material $YBa_2Cu_4O_8$ studied by Bucher *et al.* [2]. The plot is constructed purely from experimental data [2] for electrical resistivity R and nuclear spin-lattice relaxation time T_1.

between the charge carriers, which led to binding of (e) Fermion monomers into **r** space composite (2e) Bosons. These authors gave an estimate of the Boson binding energy in this underdoped cuprate studied by Bucher *et al.* [2] as $\simeq 0.01$ eV. In Ref. [3], attention was drawn to the important theoretical study of Nozières and Schmitt-Rink [4], which was published before the discovery by Bednorz and Müller [5] of the high-T_c cuprate superconductors. Nozières and Schmitt-Rink simply assumed the existence of an attractive interaction between the charge carriers and showed, by varying its magnitude from weak to strong, that one could pass smoothly from the Bardeen – Cooper – Schrieffer [6] limit to that in which precursor (2e) Bosons could begin to form in the normal state, heralding the approach of superconductivity as T was lowered to T_c, where **r** space Bosons undergo condensation. Some ideas going back to Schafroth and to Blatt [7] had the seeds of some of the results of Nozières and Schmitt-Rink. Much attention has been given recently to the theory of cross-over between the two above limits, these developments being reviewed *e.g.*, by Randeria [8].

Egorov and March [9] in later work already recognized the importance in the normal state of treating the chemical equilibrium between bound composite (2e) Bosons and Fermion (e) monomers and referred to the Law of Mass Action. Below, we shall immediately turn to introduce first a non-interacting Boson – Fermion mixture model. We shall then treat two- and three-dimensional cases in turn. Since, as will be seen, the two-dimensional case can be solved analytically, we can recover the appropriate Law of Mass Action by taking a suitable classical limit.

2. NON-INTERACTING GAS MODEL OF COMPOSITE BOSON – FERMION MIXTURE

The nature of the model we propose to discuss for the **r** space Boson – Fermion mixture is best explained by referring to Figure 2. Here two free-particle energy-wave number curves are plotted, of the usual quadratic form but characterized for Fermions by an effective mass m^* and for Bosons by an effecive mas M^*. If all the carriers are dissociated, *i.e.*, Fermion monomers, then at $T = 0$ the Fermion band,

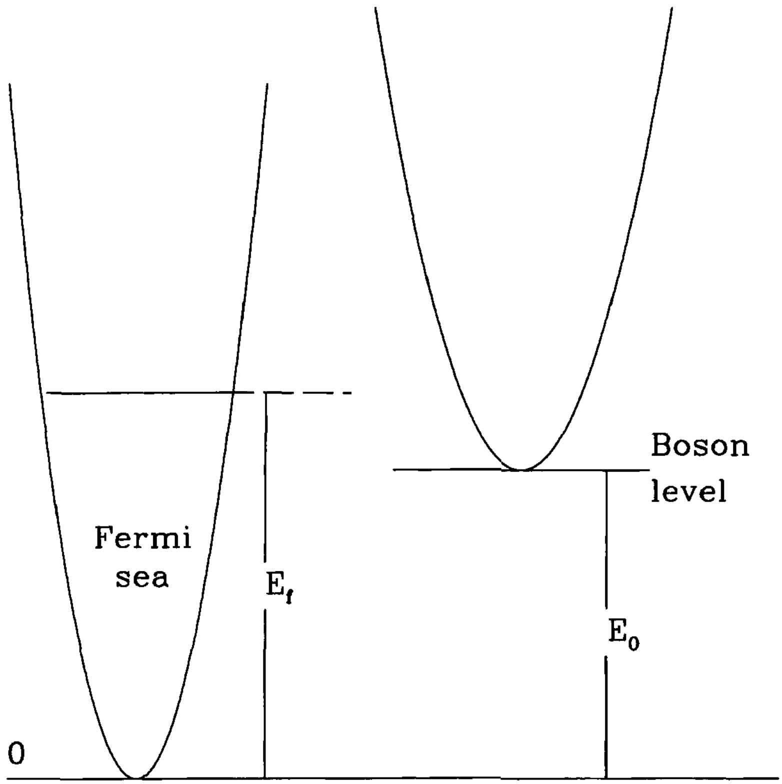

FIGURE 2 Schematic plot of the Fermion band (left) and the Boson band (right) in the present model. E_f is the Fermi energy of the fully unbound system and E_0 is the Boson level relative to the bottom of the Fermi band.

with its energy minimum taken as zero as in Figure 2, is filled up to the Fermi energy E_f. The model is then characterized by placing the minimum of the Boson dispersion relation at an energy E_0 which is positive and as drawn is less than E_f.

Let us immediately illustrate the general nature of the predictions of the model by studying the way Bosons pair as the temperature T is lowered from a value where essentially all the pairs have dissociated. Because a solution can be achieved analytically in two dimensions, it is useful to start with this example. We recall, though, that in a strictly two-dimensional system of free Bosons, Bose condensation cannot occur at any non-zero temperature.

2.1. Equilibrium Between Composite Boson and Monomer Fermion Densities in Two Dimensions

The Fermion density n_f at temperature T can immediately be written using the Fermi–Dirac distribution as

$$n_f(T) = 2\sum_{\mathbf{k}} \frac{1}{\exp[\beta(\varepsilon_{\mathbf{k}} - \mu)] + 1} \tag{1}$$

with $\beta = (k_B T)^{-1}$ and μ the chemical potential to be determined below in the mixture model. In Eq. (1) $\varepsilon_{\mathbf{k}}$ is the parabolic band shown for Fermions, namely

$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m^*} \tag{2}$$

Replacing the summation in Eq. (1) by an integration with the appropriate areal density of states in **k** space for two dimensions, the integration can be carried out to yield

$$n_f(T) = \frac{n}{2}\left(\frac{k_B T}{E_f}\right) \ln[1 + \exp(\beta\mu)] \tag{3}$$

with μ still to be determined in the model employed.

Similarly the density n_b of **r** space Bosons is given, using the Bose–Einstein distribution function, as

$$n_b(T) = \sum_{\mathbf{k}} \frac{1}{\exp[\beta(E_{\mathbf{k}} + E_0 - 2\mu)] - 1} \tag{4}$$

with $E_{\mathbf{k}}$ determined again by an equation in which m^* in Eq. (2) is replaced by the Boson effective mass M^*. Again carrying out the **k** space integration, one finds

$$n_b(T) = -\frac{n}{2}\left(\frac{M^*}{m^*}\right)\left(\frac{k_B T}{E_f}\right) \ln[1 - \exp\{\beta(2\mu - E_0)\}]. \tag{5}$$

One clearly must use, to determine μ, the fact that the total density must be given by

$$n = n_f(T) + 2n_b(T). \tag{6}$$

In presenting a numerical example of this two-dimensional model, yielding the results (3) and (5) for the Fermion and composite Boson areal densities $n_f(T)$ and $n_b(T)$ respectively, it will be useful to work with fractional quantities

$$f_f(T) = \frac{n_f(T)}{n}; \quad f_b(T) = \frac{n_b(T)}{(n/2)} \tag{7}$$

where $n_b(T)$ has been "normalized" against the total possible number of Boson pairs $n/2$. Evidently then, Eq. (6) reads

$$f_f(T) + f_b(T) = 1. \tag{8}$$

Before presenting some illustrative numerical results, we should comment on the values that we have chosen for the effective masses m^* and M^*, the total density n and the excitation energy E_0. Typical values of the band mass of carriers in high-T_c cuprates, as extracted from Photoemission Spectroscopy data as well as from band structure calculations [10], range from 2 to 5 bare electron masses m. Typical values of the number x of holes per CuO_2 unit in underdoped cuprates are in the range from $x = 0.05$ to $x = 0.1$ [10]. Using typical values of the lattice parameters in cuprates [10], these numbers correspond to total densities in the range from 3 to $7 \cdot 10^{13}\,\mathrm{cm}^{-2}$ in the two-dimensional case and from 1 to $7 \cdot 10^{20}\,\mathrm{cm}^{-3}$ in three dimensions. In our calculations we have taken $m^* = 5m$, $M^* = 2m^*$ and values of n in the ranges indicated above. For what concerns the values of the (2e) Boson excitation energy E_0, we expect them to be of the order of the energy $k_B T_m$ corresponding to the minimum of RT_1 in Figure 1 (see Section 5). We should remark that the Fermi sea state is stabilized within the present model at appreciably higher total densities of carriers than indicated above.

Figure 3 reports the fractions f_f and f_b of Fermion monomers and composite Bosons against temperature in a typical two-dimensional case. The chemical potential touches $E_0/2$ as T approaches zero, this being the signal of a Bose–Einstein condensate formed at $T_c = 0$.

In concluding this discussion of the two-dimensional version of the present non-interacting Fermion-composite Boson model, it is of

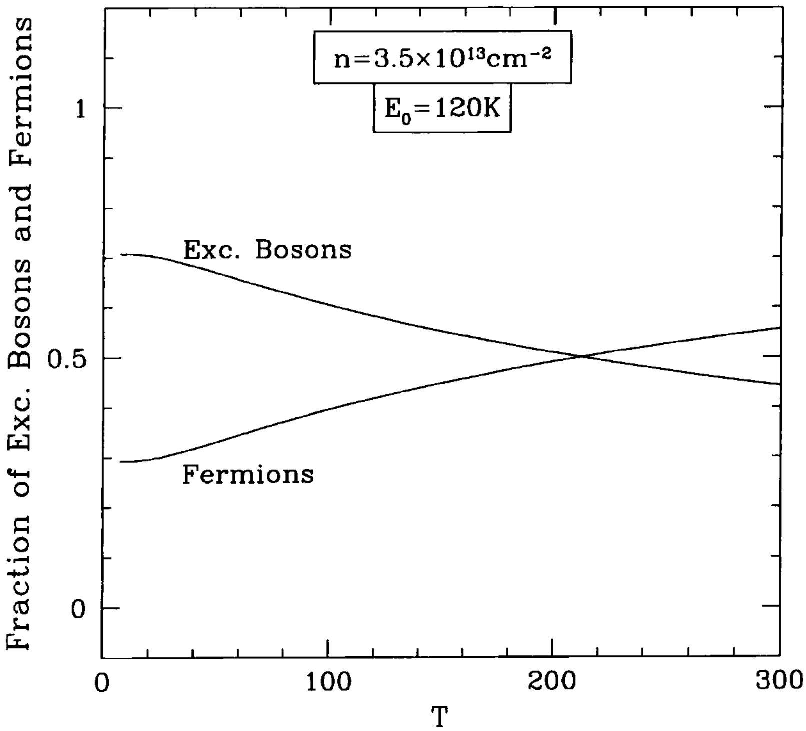

FIGURE 3 Fractions of Bosons and Fermions as functions of temperature T in the non-interacting two-dimensional model at areal density $n = 3.5 \cdot 10^{13}\,\mathrm{cm}^{-2}$ and Boson excitation energy $E_0/k_B = 120\,\mathrm{K}$.

interest to note its scaling properties. Thus, solving Eq. (3) for exp($\beta\mu$), squaring this and substituting for exp($2\beta\mu$) in Eq. (5), use of Eq. (8) readily yields the scaling property

$$f_f(T) = \left(\frac{k_B T}{E_f}\right) G\left(\frac{m^*}{M^*}, \frac{E_0}{k_B T}\right) : \tag{9}$$

i.e., $f_f(T)$ as plotted in Figure 3 depends linearly on the ratio $k_B T/E_f$, which is multiplied by a function G of just two scaled dimensionless variables, m^*/M^* and $E_0/k_B T$.

3. OVERSIMPLIFICATIONS OF THE NON-INTERACTING MIXTURE MODEL

Before going on to present results for the same non-interacting quantum-statistical mixtures model in three dimensions, it is important to point to some oversimplifications involved in it.

Notwithstanding its elegant scaling properties exhibited in Eq. (9), it seems clear to us on general physical grounds that the fraction of Bosons, $f_b(T) = 1 - f_f(T)$, should in real systems decay away more rapidly with increasing temperature than in the model results shown in Figure 3. To take an example, it can be seen there that the fraction $f_b(T)$ of Bosons remains somewhat greater than 0.3 when the thermal energy $k_B T \sim 4E_0$. One would expect that in cuprates most composite (2e) Boson pairs will by then have undergone dissociation into Fermion (e) monomers.

One reason we offer for this behaviour of the simple model of Section 2 can be seen by referring again to Figure 2. One has, effectively, placed a localized state (associated with a tightly bound composite Boson in the present picture) inside the Fermion band. But such a state could not then, in reality, have an infinite lifetime: it would be better described as a 'virtual bound state', with an energy width ΔE, and hence from the Uncertainty Principle a finite lifetime $\sim \hbar/\Delta E$. Similar considerations apply, of course, to thermally excited composite Bosons, whose dissolution into Fermion monomers as temperature increases will be favoured by a finite lifetime.

Of course, in two dimensions, as Figure 3 makes plain, there is no Bose condensation in the non-interacting model at any non-zero temperature. It is indeed already known from experiment that, in the high temperature superconducting cuprates, T_c is indeed sensitive to "dimensionality" in that it strongly depends on interlayer coupling between CuO_2 planes [10].

Bearing in mind the above comments, we turn to present results for the model of Section 2 in three dimensions. We stress, however, that the results presented are for an isotropic model while as indicated above the high-T_c cuprates have pronounced anisotropy. Nevertheless, some interesting features still emerge from this oversimplified, now three-dimensional, model.

4. PREDICTIONS OF ISOTROPIC THREE-DIMENSIONAL VERSION OF NON-INTERACTING MIXTURES MODEL

In three dimensions, after replacement of summation over **k** by integration in Eqs. (1) and (4), one must proceed by purely numerical methods to evaluate $f_f(T)$ and $f_b(T)$, the fractions of Fermion monomers and composite Bosons respectively.

However, the new feature to be emphasized is that, in three dimensions, in this simple model the **r** space Bosons condense at a non-zero temperature T_c. This can be obtained in the simple Fermion–Boson mixture model by approaching T_c from high temperatures. Eq. (6) now changes to

$$n = n_f(T) + 2n_c(T) + 2n_{b,\,\mathrm{exc}}(T), \tag{10}$$

where n_c is the density of Bosons in the condensate and $n_{b,\,\mathrm{exc}}$ is the density of Bosons out of the condensate and is given by Eq. (4) when the $\mathbf{k} = 0$ term is taken out. The critical temperature T_c is determined from Eq. (10) as the temperature at which the chemical potential μ equals $E_0/2$. Below T_c, μ is kept constant and Eq. (10) is used together with Eqs. (1) and (4) to evaluate $n_c(T)$.

Figure 4 shows now an analogous plot in three dimensions to that presented in Figure 3 for the two-dimensional model. Above T_c, which is used to scale the temperature T in the plot, for the parameters shown, the behaviour of f_f and f_b strongly resembles the two-dimensional behaviour, with again very slow decay with temperature of the Boson density. Again, as in Figure 3, there is an intersection point: $f_b = f_f = 1/2$, at temperature T_x say.

The behaviour of this temperature T_x, relative to T_c, is explored as a function of the model parameters n and E_0 in Figure 5. For densities beyond about $2 \cdot 10^{20}\,\mathrm{cm}^{-3}$ there is relative insensitivity of T_x/T_c to parameter variation, the range being such that $5 < T_x/T_c < 5.8$. The comments made on Figure 3 already in two dimensions such that these values of T_x/T_c are rather too high are still relevant: an artifact of the failure of the model to allow Boson states within the Fermion band to have a finite lifetime.

The physical meaning which underlies all the curves in Figure 5 is the dependence of the crossing temperature T_x on the energy

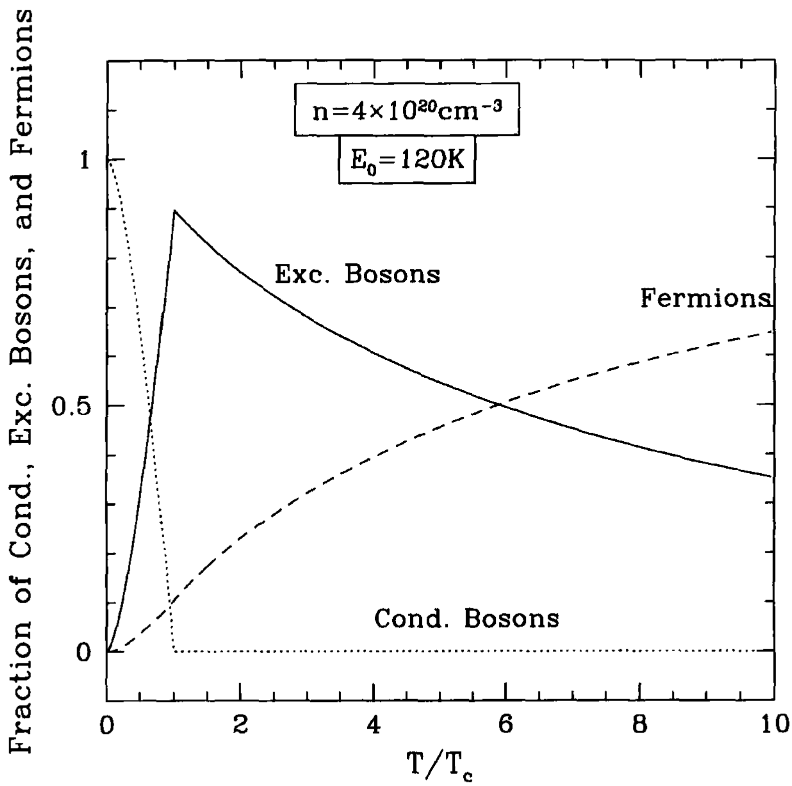

FIGURE 4 Fractions of condensed Bosons (dotted line), excited Bosons (solid line) and Fermions (dashed line) as functions of reduced temperature T/T_c in the non-interacting three-dimensional model at density $n = 4 \cdot 10^{20}\,\mathrm{cm}^{-2}$ and Boson excitation energy $E_0/k_B = 120\,\mathrm{K}$.

$E_b = 2E_f - E_0$, which may be looked upon as a binding energy for the first composite Boson. Decreasing E_b will favour dissociation of Boson dimers into Fermion monomers. One may decrease E_b either by decreasing the Fermi energy (through a decrease of the total density) at fixed E_0, or by increasing E_0 at fixed E_f. A similar behaviour is found in the two-dimensional case, even though we have not explicitly shown it here.

To press the points made above, and also to comment on the model predictions in the regime of $T < T_c$, we return to contact with experiment, which can first of all be made *via* Figure 1.

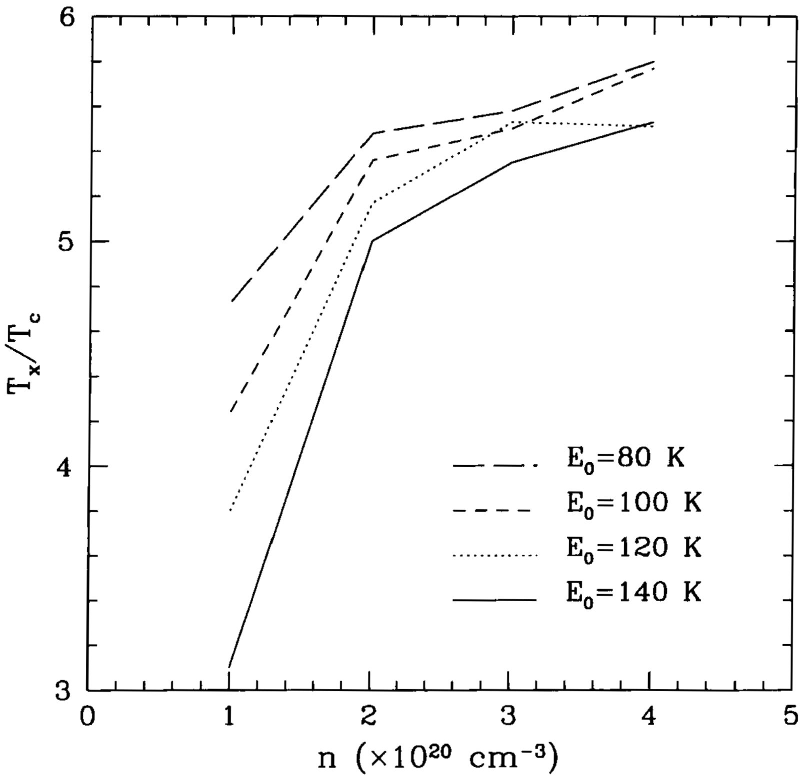

FIGURE 5 Crossing temperature T_x scaled with the transition temperature T_c is shown as a function of density for different values of the Boson excitation energy (E_0/k_B = 80, 100, 120 and 140 K) corresponding to the temperatures near the minimum in the RT_1 plot of Figure 1.

5. FURTHER CONTACT WITH EXPERIMENT

5.1. Normal State Properties

Let us return to the plot of RT_1 *vs* T [1] reproduced in Figure 1. Assuming the apparent linearity at the highest temperatures shown to be associated with dominantly fully dissociated Fermion monomers, it is clear that the present picture must view the turn-up of the curve starting at its minimum T_m as due to the formation of a substantial number of **r** space composite Bosons.

It is worthy of note that it is mainly T_1 that is sensitive to the formation of precursor Bosons. R, in contrast, remains approximately proportional to T^2. We interpret this as due, in the formula $\sigma = R^{-1} = n_{\text{carrier}} q^2 \tau / m^{\text{eff}}$, to a balance between the density n_{carrier}, the squared charge q^2 and the effective mass m_{eff} for (e) Fermion monomers and charged (2e) Bosons, though the argument depends on a combination of reciprocal relaxation times not being markedly different for Boson pairs and Fermion monomers.

While, of course, any focus on the 'intersection' temperature T_x introduced above consists of a 'criterion' to grossly characterize the two normal state densities $n_f(T_x)$ and $n_b(T_x)$, where in fact half the total available Fermion monomers are now bound in pairs, one must expect that at T_x the **r** space Bosons will substantially influence normal state physical properties. Thus, one can argue plausibly that the temperature T_m corresponding to the minimum of Figure 1 is $\sim T_x$. However, for reasons already discussed, we do expect the simple non-interacting mixture model of Section 2 to overestimate T_x/T_c and that appears to be borne out by comparison of the model predictions with the minimum in Figure 1.

5.2. Behaviour Below Superconducting Transition Temperature T_c

In earlier work, Egorov and March [9] used experiments on transverse plasmons [11] to extract Boson densities. Their points for $T < T_c$ are plotted as the solid circles in Figure 6.

We have assumed that their data, which they normalized actually to the Boson density at 8 K, can be roughly compared with the Boson condensate density (see also Fig. 4), and this is plotted from the simple model of Section 2 in the continuous curve shown in Figure 6, for $n = 4 \cdot 10^{20}\,\text{cm}^{-3}$ and $E_0 = 120\,\text{K}$. There is some gross similarity of shape but quantitative agreement is lacking. We shall return to this point in Section 6 below.

6. SUMMARY AND CONCLUSION

The non-interacting Boson–Fermion mixture model proposed in Section 2 to treat the equilibrium between **r** space composite Bosons

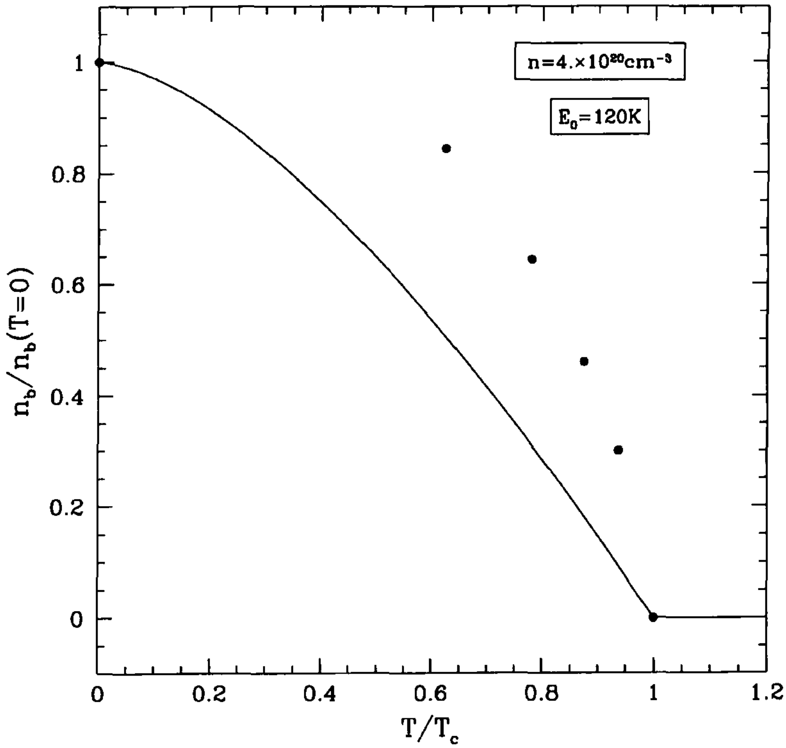

FIGURE 6 Solid circles are experimental data (from Ref. [11]) of the transverse plasmon frequency scaled with its value at $T = 8$ K as a function of reduced temperature T/T_c. The solid line is the condensate fraction calculated within the non-interacting three-dimensional model using the same parameters as in Figure 4.

and Fermion monomers supports the interpretation of the data plotted in Figure 1 as heralding the approach of the superconducting transition in this underdoped cuprate studied by Bucher *et al.* [2] by the formation of precursor **r** space composite Bosons. We have shown that the model is consistent with Bosons beginning to have a clear impact on physical properties in the normal state at temperatures appreciably above T_c. We feel, however, that the 'free' Bosons and Fermions assumed in the model of Section 2 will overestimate somewhat the fraction of Bosons in the tail of the normal state density. Hence future refinement of the model proposed here, should these be motivated by relevant experiments, will have to take cognizance of the finite lifetime of the composite Bosons and of the screening associated with charged particles constituting the real Boson – Fermion mixture.

While, of course in parallel with the theoretical work of Nozières and Schmitt-Rink [4], it has not proved necessary to assume any particular mechanism for the attractive interaction leading to precursor composite Boson formation in the normal state, the present investigation leaves little doubt that on the underdoped cuprate focused on in this work, we are well away from the weak coupling BCS regime of superconductivity.

Following this summary, we briefly return to two aspects of the present study where we feel further work may be fruitful in the future. High-T_c cuprates are, without question, highly anisotropic. We have attempted, in dealing with the non-interacting model at the heart of the present study, to reflect this point by comparing and contrasting two and three dimensions. We have concluded that, in a more refined study, both anisotropy (related to dimensionality) and interactions will require inclusion. Along these lines, relevant work is that of Ranninger and Robaskiewicz [12] and Friedberg and Lee [13 – 14], whose model includes, *via* a parameter, Boson – Fermion interaction. Such interaction will be seen in the appendix to have a correct influence on quantitative details of the present non-interacting model.

Acknowledgements

One of us (N. H. M.) wishes to thank the Scuola Normale Superiore for its generous hospitality during his visit, in which his contribution to this work was brought to fruition. He also wishes to thank Professor R. Pucci for numerous valuable discussions on the area embraced by this article and Mr. G. Angilella for valuable discussions on dimer-monomer chemical equilibrium. M. L. C. would like to thank the Istituto Nazionale per la Fisica della Materia (INFM) through the Advanced Research Project PRA/BEC.

References

[1] Egorov, S. A. and March, N. H. (1994). *Phys. Chem. Liquids*, **27**, 195.
[2] Bucher, B., Steiner, P., Karpinski, J., Kaldis, E. and Wachter, P. (1993). *Phys. Rev. Lett.*, **70**, 2012.

[3] March, N. H., Pucci, R. and Egorov, S. A. (1994). *Phys. Chem. Liquids*, **28**, 141.
[4] Nozières, P. and Schmitt-Rink, S. (1985). *J. Low Temp. Phys.*, **59**, 195.
[5] Bednorz, J. G. and Müller, K. A. Z. (1986). *Z. Phys.*, **B64**, 189.
[6] See, for example, Schrieffer, J. R. Superconductivity, Benjamin, Reading, 1964.
[7] Schafroth, M. R., Butler, S. T. and Blatt, J. M. (1957). *Helv. Phys. Acta*, **30**, 93; Blatt, J. H. (1964). Theory of Superconductivity, Academic Press, New York.
[8] Randeria, M. In: Griffin, A., Snoke, D. W. and Stringari, S. (1995). Bose-Einstein Condensation, Cambridge University Press, Cambridge, p. 355; Randeria, M. in: Schrieffer, R. J., Iadonisi, G. and Chiofalo, M. L. *Proc. Int. School of Physics* "Enrico Fermi", Course CXXXVI, Models and Phenomenology for Conventional and High-Temperature Superconductivity, to be published in 1998.
[9] Egorov, S. A. and March, N. H. (1995). *Phys. Chem. Liquids*, **30**, 59.
[10] See for example, Burns, G. High-Temperature Superconductivity. An Introduction, Academic Press, Boston 1992, and references given therein.
[11] Tamasaku, K., Nakamura, Y. and Uchida, S. (1992). *Phys. Rev. Lett.*, **69**, 1455.
[12] Ranninger, J. and Robaskiewicz, S. (1985). *Physica*, **135**, 468. See also Ranninger, J. and Robin, J.-M. (1997). *Phys. Rev.*, **B56**, 8330.
[13] Friedberg, R. and Lee, T. D. (1989). *Phys. Rev.*, **B40**, 6745. See also: Bleer, A. S., Ren, H. C. and Tchernyshyov, O. (1997). *Phys. Rev.*, **B55**, 6035.
[14] For the model in a layered structure, Friedberg, R., Ren, H. C. and Lee, T. D. (1991). *Phys. Lett.*, **A152**, 417.
[15] Iadonisi, G., Cataudella, V., Ninno, D. and Chiofalo, M. L. (1995). *Phys. Lett.*, **A196**, 359.
[16] Piegari, E., Cataudella, V. and Iadonisi, G., *Physica, C.*, in press.
[17] Chiofalo, M. L. *Ph.D. Thesis*, Scuola Normale Superiore, Pisa, 1997.

A. INTERACTING MODEL

In this Appendix we explicitly show that inclusion of a Boson–Fermion interaction as in Ref. [13] reduces T_x/T_c. While referring to the original papers [12–13] for the details of the model, we report here how Eq. (6) is modified by the introduction of a coupling strength parameter g, which drives the dissociation of a composite Boson into two Fermion monomers. The model Hamiltonian [12–13] is

$$\begin{aligned} H = & \sum_{\mathbf{Q}} \left(E_0 + \frac{\hbar^2 Q^2}{2M^*} - 2\mu \right) b^\dagger_{\mathbf{Q}} b_{\mathbf{Q}} \\ & + \sum_{\mathbf{k}} \left(\frac{\hbar^2 k^2}{2m^*} - \mu \right) c^\dagger_{\mathbf{k}} c_{\mathbf{k}} \\ & + g \sum_{\mathbf{Q},\mathbf{k}} (b^\dagger_{\mathbf{Q}} c_{\frac{\mathbf{Q}}{2}+\mathbf{k},\uparrow} c_{\frac{\mathbf{Q}}{2}-\mathbf{k},\downarrow} + h.c.), \end{aligned} \tag{11}$$

$c^{(\dagger)}$ and $b^{(\dagger)}$ being the destruction (creation) operators for the Fermions and Bosons respectively.

By the replacement $b_0^\dagger b_0 = n_c$, where n_c is the condensate fraction, and retaining only the terms with $\boldsymbol{Q} = 0$ in the last term of Eq. (11), the Hamiltonian (11) can be diagonalized [12–13]. The resulting renormalized Fermion spectrum is

$$E(k) = \left[\left(\frac{\hbar^2 k^2}{2m^*} - \mu\right)^2 + g^2 n_c\right]^{\frac{1}{2}}, \tag{12}$$

whereas the renormalized Boson excitation energy ν is

$$\nu = E_0 + \frac{g^2}{2}\sum_{\mathbf{k}} \mathcal{P}\frac{1}{\nu - k^2/2m^*} \tag{13}$$

Within the grand-canonical ensemble it is possible to determine the macroscopic occupation number n_c and the chemical potential μ by requiring that the free energy is a minimum [12–13] and that the total number of particles is conserved. One obtains thereby the set of coupled equations

$$\mu = \frac{E_0}{2} - \sum_{\mathbf{k}} \frac{g^2}{E(k)} \tanh\frac{\beta E(k)}{2} \tag{14}$$

and

$$n = 2n_c + 2n_{b,\mathrm{exc}} + n_f, \tag{15}$$

where

$$n_{b,\mathrm{exc}} = \sum_{\mathbf{k}} \frac{1}{e^{\beta\left(\nu + \frac{\hbar^2 k^2}{2M^*} - 2\mu\right)} - 1} \tag{16}$$

and

$$n_f = \sum_{\mathbf{k}} \frac{F(k)}{E(k)\left(1 + e^{-\beta E(k)}\right)} \tag{17}$$

with

$$F(k) = \left[E(k) + \mu - \frac{\hbar^2 k^2}{2m^*} + \left(E(k) - \mu + \frac{\hbar^2 k^2}{2m^*}\right) e^{-\beta E(k)}\right]. \tag{18}$$

Eq. (14) has the meaning of a "gap" equation and follows from minimization of the free energy with respect to n_c, while Eqs. (15)–(18) are analogous to Eqs. (10), (1) and (4) as modified by the presence of the coupling g. The non-interacting mixture model is recovered by setting $g = 0$.

Equations (14)–(18) can be solved self-consistently to determine μ, n_c, $n_{b,\,\mathrm{exc}}$ and n_f as functions of the temperature T for a given total density n. The critical temperature is obtained by setting $n_c = 0$ and solving the coupled equations for T and μ at density n. Above T_c Eq. (14) has to be dropped.

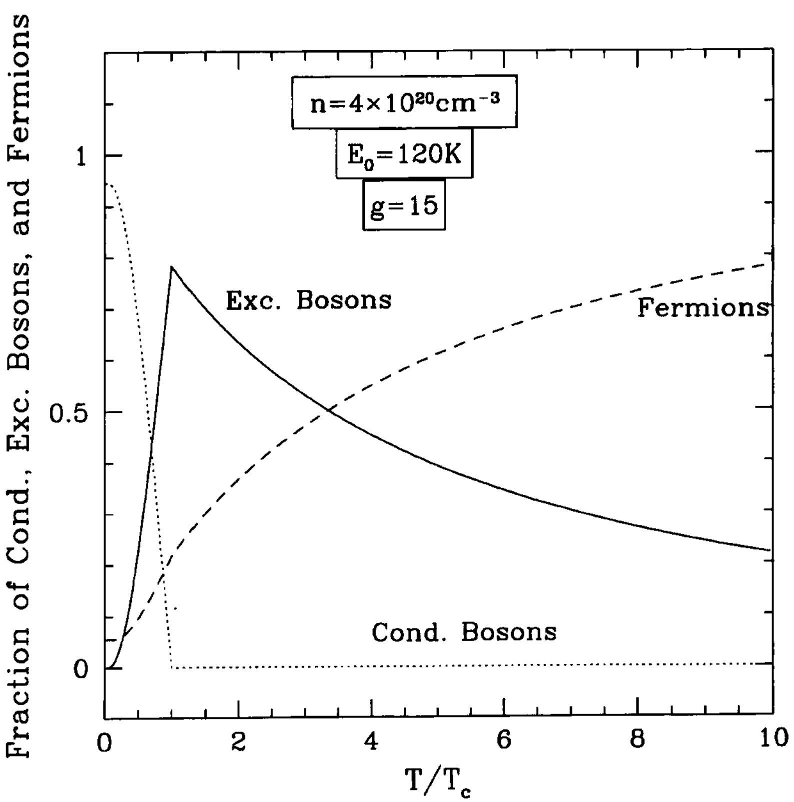

FIGURE 7 Fractions of condensed Bosons (dotted line), excited Bosons (solid line) and Fermions (dashed line) as functions of reduced temperature T/T_c in an interacting three-dimensional model. The parameters are the same as in Figure 4 and the interaction strength is $g = 15$.

Equations (14–18) have been solved numerically both below and above the transition temperature [15–16]. Here we show the numerical solution obtained by using the unrenormalized Boson excitation energy E_0 in place of ν. The results are reported in Figure 7 for the same parameters as in Figure 4 and for $g = 15$. The crossing temperature T_x is reduced with respect to the non-interacting case, although the reduction is expected to be still underestimated. Firstly, in the above approximate solution Bosons with $Q \neq 0$ still have an infinite lifetime against dissociation into Fermions; secondly, we have not taken into account the renormalization of the excitation Boson energy as implied by Eq. (13). In Figure 7, the shift of T_x/T_c relative to the results of the non-interacting model in Figure 4 is mainly due to the destabilization of the composite Bosons relative to Fermions below T_c. As a consequence, the fraction of excited Bosons at T_c is depressed and the dissociation into Fermion monomers is enhanced.

As a final comment, we may point out that, relative to the results shown in Figure 6, the above interacting model also yields an improved account of the condensate density as obtained from measurements of the London penetration depth [17].